# Fault Detection Combining PCA, Control Charts and Statistic Operation Limits

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**Abstract.** As processes grow in size and complexity, the monitoring of them becomes more important, to avoid production losses and even accidents involving damage to personnel health and equipments. Process monitoring is particularly challenging due to the presence of both continuous and discrete variables, noisy information and lack or excess of data. There are two important tasks involved in the process monitoring: fault detection and fault diagnosis. This paper proposes a fault detection framework combining Principal Components Analysis (PCA), Control Charts and a comparison with Statistic Limits obtained from historical data process and inductive learning. PCA and control charts have been used in the past to detect suspicious observations. Once the suspicious observations are detected, a contribution chart and a comparison with the statistic limits are performed for fault detection. We show preliminary results from an electric circuit simulation composed by five subsystems.

Keywords: Fault Detection, Principal Component Analysis, Inductive Learning, Control Charts, Statistic Limits.

## 1 Introduction

As processes become more complex, the monitoring of them is very important in order to improve process performance, efficiency and product quality. Monitoring of industrial processes plays a substantial role in system safety, availability and production quality. Early detection of faults can help to avoid major breakdowns and incidents. In order to tackle those problems, fault detection and system diagnosis has been an active research domain since a few years ago.
There exist many research works related with fault detection. Most of the methods used are analytic, based on artificial intelligence (AI) or statistical methods.

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[1] classifies fault detection and isolation methods in three groups. 1) Quantitative Model Based, 2) Qualitative Model Based and 3) Process History Based.
Quantitative Model Based fault detection methods are based on a mathematical model of the system. The occurrence of a fault is capture by discrepancies between the observed behavior and the prediction made by the model. These approaches use state estimation, parameter identification techniques, and parity relations to generate residuals. Fault localization then, rests on interlining the groups of components that are involved in each of the detected discrepancies. However, it is often difficult and a time-consuming task to develop accurate mathematical models that characterize all the physical phenomena occurring in industrial processes.
Qualitative Model based fault detection methods use symbolic reasoning which generally combines different sources of knowledge with graph theory to analyze the relationships between variables of a system. An advantage of these methods is that an explicit model of the system to be diagnosed is not necessary. Knowledge-based approaches such as expert systems, may be considered as alternative or complementary approaches where analytical models are not available.
Process History Based fault detection methods only require a big quantity of historical process data. There are several ways in which these data can be transformed and presented as prior knowledge of a system. These transformations are known as feature extraction and could be qualitative, as those used by expert systems, and qualitative trend analysis methods or quantitative, as those used in neural networks, PCA, PLS or statistical pattern recognition.
There are many papers related to fault detection and diagnosis in different processes. They use either an individual technique or make a combination of different techniques, taking advantage of the best characteristics of each method, to perform a better general behavior of the fault detection process. [2] presents a comparative study in the monitoring of hybrid systems, where the continuous part is modeled by Bond Graph and the discrete part is modeled by Petri Nets. [3] proposes a structure of a hybrid fault diagnosis system which integrates Signed Directed Graph, Artificial Neural Networks and dynamic simulation. [4] introduces a method based on hybrid system theory, which combines knowledge base methods and model base methods. [5] proposes a process monitoring which is composed of three parts: preanalysis, visualization and diagnosis, where the proposed method integrates PCA, FDA and clustering analysis taking advantage of each technique for a complete solution. [6] describes plant devices, sensors, actuators and diagnostic tests as stochastic finite state machines, by assigning transition probabilities and marginal probabilities to safe and fault events. By using simple composition rules, it is possible to determine the feasible configuration of alarms and their conditional probability given any event. [7] combines the use of signed directed graph to make a classification model, PCA and fuzzy knowledge to form a qualitative and quantitative model and compares the grade of the patterns needed to be diagnosed to the given fault patterns. [8] proposses a method based on the interaction between AI and control techniques. It uses a causal graph representation of the process, enabling decomposition into sub-

systems and reducing the diagnostic computational complexity. After that, at local level, FDI techniques based on numerical residual generation and analysis are carried out. [9] proposes a useful method when measures on the input signals can not be done due a nonexistent sensor or because it is impossible to do measurements due to the nature of the system itself. Thus it takes plant output signals, combines its variances, and uses a discriminant analysis upon the resultant features to carry out the diagnosis. In [10] PCA and statistical control charts are used to detect process operating faults on an industrial rolling mill reheating furnace. The Q statistic and Hotelling $T^2$ statistic are used to calculate the control limits of the statistical control chart. [11] proposes a fault diagnosis model based on machine learning which extracts multi-dimension features from the detected signal to supervise the different features of it simultaneously.
In this paper we propose a fault detection framework combining Principal Components Analysis (PCA), Control Charts and a comparison with Statistic Limits obtained from historical data process and inductive learning. We show preliminary results from an electric circuit simulation composed with five subsystems. The organization of the paper is as follows: section 2 gives preliminaries which explaines how the statistical limits are obtained as well as background knowledge on PCA and control charts. Section 3 gives the framework general description. Section 4 shows how the framework works in a simulation example with single and multiple faults and the performance of it in presence of white noise on measurements, as well as a comparison of the general performance of it against two similar frameworks. Section 5 gives conclusions to the paper.

# 2 Preliminary

## 2.1 Automatic Statistical Limits Obtention

[11] gives an algorithm to extract the statistical boundary vectors of a multi-dimensional feature extraction. In this paper a modification of that algorithm is done. Instead of doing multi-dimension feature extraction, here we work just with the statistical mean of the system variable being measured. Thus, the algorithm of inductive learning is used to obtain the statistical boundary vectors of $w_{max}(t_1, t_n, i)$ and $w_{min}(t_1, t_n, i)$ from a matrix which $m_i$ rows are the different subsystems forming an entire process and the $n$ columns present the changes of the statistical mean of the subsystem variable being measured as time changes from time $t_1$ to time $t_n$. The algorithm is shown below.

1. Initialize the statistical boundary vectors $w_{max}(t_1, t_n, i) = [w(t_1, t_2, 1) \cdots w(t_1, t_2, m_i)]^T$, $w_{min}(t_1, t_n, i) = [w(t_1, t_2, 1) \cdots w(t_1, t_2, m_i)]^T$ and the counter $j = 0$.
2. Calculate $w_{max}(i) = [max(w_{max}(t_1, t_n, 1), w_{max}(t_n, t_{n+1}, 1) \cdots max(w_{max}(t_1, t_n, m_i), w_{max}(t_n, t_{n+1}, m_i)))]^T$ and calculate $w_{min}(i) = [min(w_{min}(t_1, t_n, 1), w_{min}(t_n, t_{n+1}, 1) \cdots min(w_{min}(t_1, t_n, m_i), w_{min}(t_n, t_{n+1}, m_i)))]^T$. If $w_{max}(i) = w_{max}(t_i, t_n, i)$ and $w_{min}(i) = w_{min}(t_i, t_n, i)$, then $j = j + 1$, else $j = 0$.

3. If $j \geq V_1$, go to step(4), else $w_{max}(t_1, t_n, i) = w_{max}(i)$, $t_n = t_{n+1}$, go to step (2)
4. Output $w_{max}(t_1, t_n, i)$, $w_{min}(t_1, t_n, i)$, $t_n$, exit.

Where $w_{max}(t_1, t_n, i)$ and $w_{min}(t_1, t_n, i)$ in the present paper are used as the desired statistical limits for the statistical mean of the system variable being measured.

### 2.2 Principal Components Analysis (PCA)

The principal component analysis is concerned with explaining the variance and covariance structure of a set of variables, through a few linear combinations of these variables. The general objectives are basically: data reduction and interpretation.

PCA decomposes the $X$ original data matrix with dimension $m \times n$ ($m$ number of samples and $n$ number of variables) as:

$$X = t_1 p_1^T + t_2 p_2^T + \cdots + t_n p_n^T + E = T_n P_n^T + E \tag{1}$$

$t_i$ vectors are called the scores of the principal components and have information on how the samples are related to each other. $p_i$ vectors are the eigenvectors of the covariance matrix of $X$, and are known as the loads of the principal components. They have information on how the variables are related to each other. In fact principal components analysis splits $X$ matix in two parts, one that describes the system variation and other one that captures noise or information not modeled. The $X$ matrix could often be aproximated using only $A$ ($\leq n$) principal components instead of $n$ variables as

$$\hat{X} = \sum_{i=1}^{A} t_i p_i^T + e \tag{2}$$

Where $e$ is the residual. PCA is scale dependent, thus when variables are measured in different scales or on a common scale with widely differing ranges, they are often standardized. Another important issue is the minimum quantity of components needed to explain the data. The number of PC to retain in order to represent the maximum variance depends on the data and the existing correlation between the variables such that there are several decision criteria. [12] proposes to consider the amount of total sample variance explained, the relative sizes of eigenvalues or the use of scree plots. Thus the number of principal components should be equal or less than the variables of $X$. When the maximum variance of data is explained with the first two principal components, samples lie on a plane and a constant density ellipse could be formed by them.

Figure 1 shows the plane and constant density ellipse formed by two principal components, where the first principal component is the one that has the major data variation, while the second one is the next with the major data variation of the rest and is orthogonal to the first one. Thus, PCA model is able to describe significant variations in a fewer dimension than the original $n$ variables does.

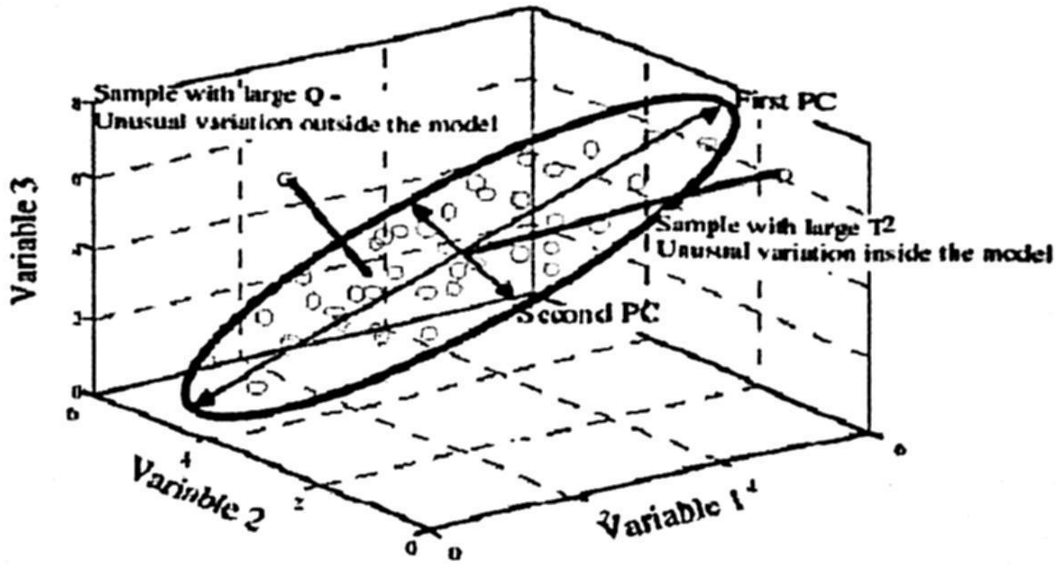


**Fig. 1.** Plane and constant density ellipse formed by two principal components.

### 2.3 Control and Contribution Charts in Statistical Process Control

Generally, there are two statistics to define the action and warnings limits used in multivariate control charts. The first statistic is Hotelling's $T^2$ as follows:

$$T_i^2 = X_i P \Lambda P^T X_i^T \sim \frac{A(n-1)}{n-A} F_{A,n-A} \tag{3}$$

Where $X_i$ is the vector containing the data matrix $X$ at sample time $i$, and $\Lambda$ is a diagonal matrix containing the inverse of the eigenvalues of the PC scores. $T^2$ is a statistical measure of the multivariate distance of each observation from the center of the data set. This is an analytical way to find the most extreme points in the data. Thus, an out of control signal is identified if

$$T_i^2 > \frac{A(n-1)}{(n-A)} F_{A,n-A,\alpha} \tag{4}$$

confidence limit $\alpha$ typically takes the value of 0.05 or 0.01 for the limits. The second metric used in process monitoring to identify non-conforming operation is the $Q$ statistic (also referred as Squared Prediction Error, SPE). The $Q$ statistic is defined to be the quadratic form of the residuals, that is the squared difference between the observed values and predicted values from the nominal or reference models:

$$Q_i = e_i e_i^T = \sum_{j=1}^{k} (x_{ij} - \hat{x}_{ij})^2 \tag{5}$$

And its upper limit (UL) is given by a chi-square distribution with $p-A$ degrees of freedom

$$UL = \chi^2_{p-A}(\alpha) \tag{6}$$

$Q$ is the statistic that measures lack of fit of a model to data. Under the assumption that the linear PCA is valid, the $Q$ statistic defines the Euclidean distance of the position of an observation from the hyperplane formed by the PCA model. See figure 1. In addition we could determine which variable of the process being analyzed is responsible for the unusual $Q$ behavior, looking at a chart showing the contribution of each input to the $Q$ statistic. This chart is known as the contribution chart and includes all process variables and their corresponding PCA scores in its axis.

## 3 Framework Description

The proposed detection framework is shown in figure 2. As the framework is a Process History Based fault detection method, this only requires a big quantity of historical data process. This data set takes into account only the normal system data operation. They will be transformed by both, PCA model and the normal operation data limits, and used as prior knowledge of the system to perform the detection process.

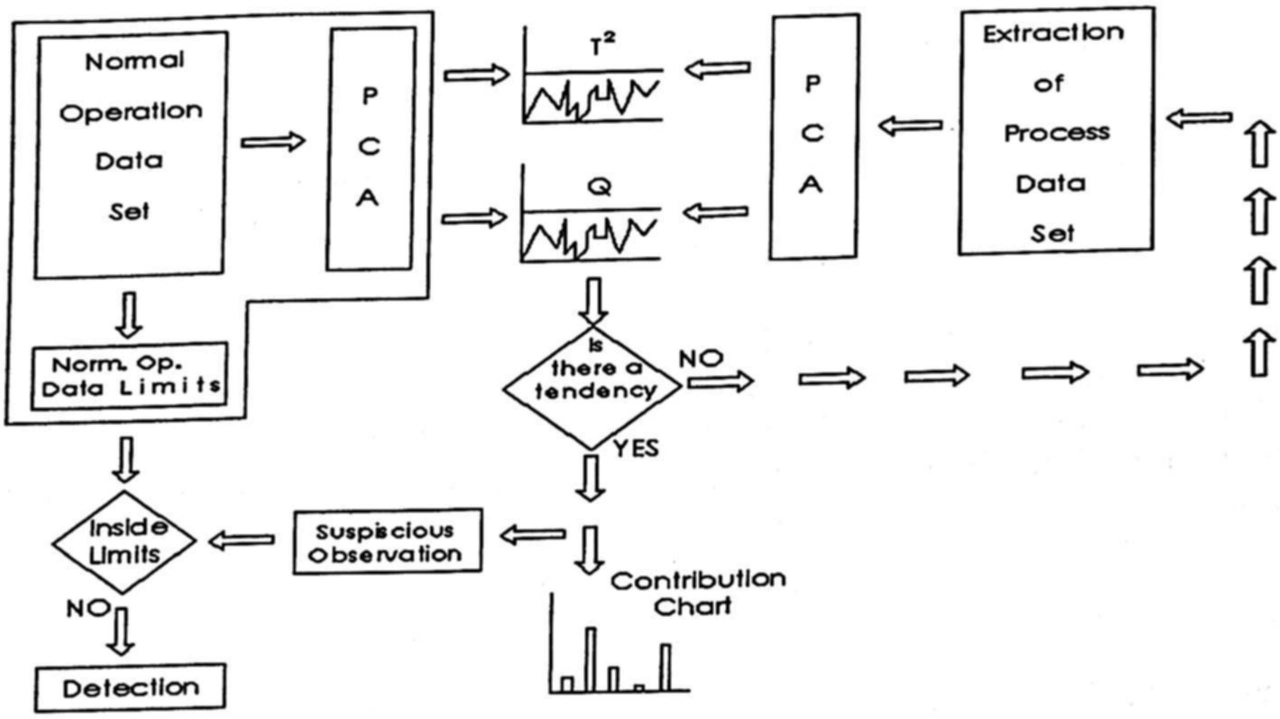


**Fig. 2.** General fault detection framework

The first step is to obtain a normal operation data set from system or process and a standardization procedure is carried out. From this data set a PCA model is obtained to see the relationships and to find out the correlations between variables, that at a first sight would be very difficult to notice. Then, a decrement in original data matrix dimension is achieved, allowing to work with less but enough data to describe the maximum variability of original system. The PCA model gives the loads and scores of the principal components corresponding to normal operation. With this loads and scores of normal operation, the value of the explained variance for each component as well as the boundaries for Hotelling's $T^2$ and $Q$ statistics are obtained (equations 4 and 6 respectively). They are the corresponding limits in control charts. In addition, the normal operation data limits are obtained following the steps mentioned in section 2.1. An adequation to the method described in [11] is being done. Instead of taking several signal transformation functions (STF) and so many single output functions (SOF), here it is taken just one STF and one SOF. Raw data is taken as the STF and the mean as the SOF, making it possible to obtain the minimum and maximum statistic limits that variables being monitored should have in normal operation.

For detection process, a process data set is extracted and analized as follows. A PCA model is built from the extracted process data set, and the scores of the principal components given by this model are plotted in control charts comparing them with the $T^2$ and $Q$ statistics corresponding to normal operation model. If the chart does not exhibits a trend, another process data set is extracted to be analized. But if a normal operation $T^2$ or $Q$ limit is violated or a trend is present, two actions are taken. 1)A contribution chart is done to find out which variable or variables has the major contribution to system's variability, and 2)the suspiscious observation is compared to the normal operation data limits previously obtained to verify if it is out of bounds.

## 4 Case Study

This section shows the performance of our framework in a simulation example. The simulation consists in the operation of an electric system formed by five subsystems. Each subsystem is simulated with different RL series circuits (see figure 3). A change within $\pm 10\%$ of the original values in each subsystem's components is considered as normal operation. An electrical current sensor is available such that each subsystem's current could be measured. After PCA a reduction to 2 variables was obtained.

The methodology proposed is applied as follows:

1. From normal operation history process data (electrical current in each subsystem), build PCA model and obtain $T^2$ and $Q$ statistics as well as the minimum and maximum limits for each subsystem's current.
2. Take a test data set.
3. Build a PCA to the test data set and obtain a reduction on original dimensions.
4. Build and observe control charts for $T^2$ and $Q$ statistics. If control chart detects a trend in a specific time instant go to 5), else go back to 2).
5. Build contribution chart and obtain the electrical current value for suspiscious time instant (sample).
6. Compare the suspiscious electrical current value with its normal operation limits obtained in 1) and detect which subsystem is in faulty mode.

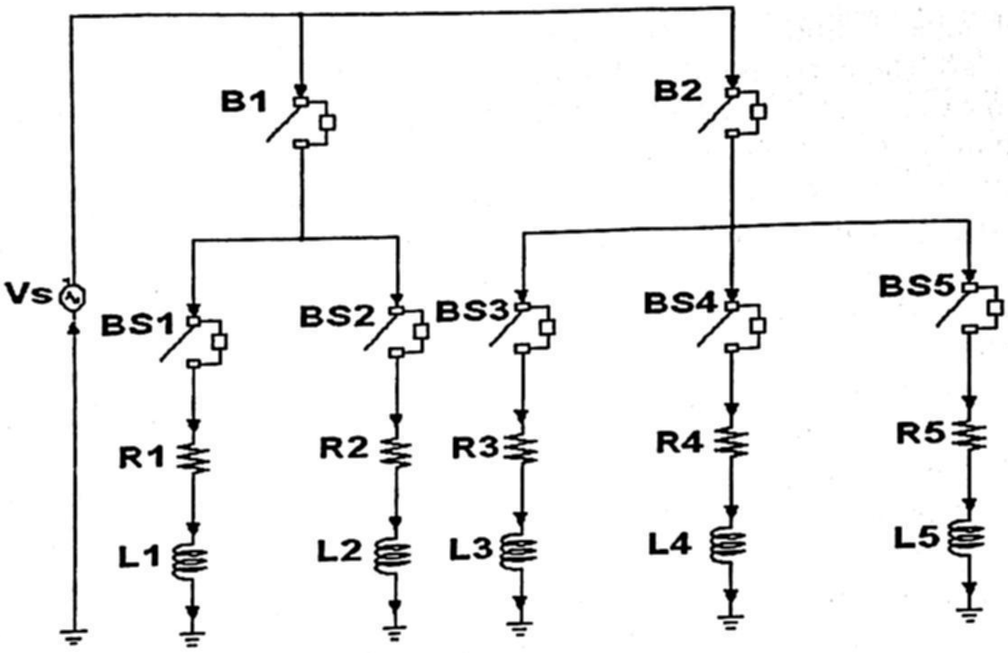


**Fig. 3.** Simulation of a system formed by five subsystems.

### 4.1 Single Fault

Several runs have been done to detect simple faults and obtain the effectiveness percentage in this task. A ramp input, a step input and a combination of both were simulated in different subsystems to see the performance of the methodology proposed. For instance, a simulation of a single fault present in subsystem 2 in which current decrements in steps of 0.9% is included in sample 80. Figure 4 shows how control charts depict a trend to pass its control limits.

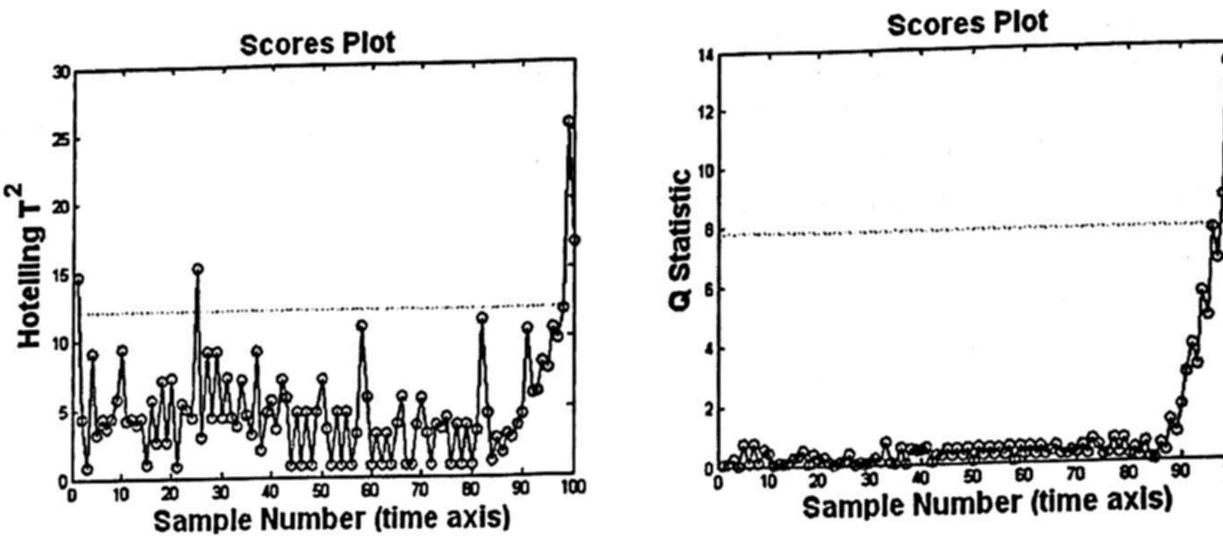


**Fig. 4.** Control charts for a decrement of 0.9% in subsystem's 2 current.

Figure 5 shows that variable 2 (subsystem 2) is the one that has the major variability of the system, indicating thus that subsystem 2 is probably in faulty mode. Then, when checking this suspicious observation against its corresponding normal operation limits, it is found that subsystem's 2 electrical current value has decreased under the lower current normal operation limit, having in this way detected the fault.

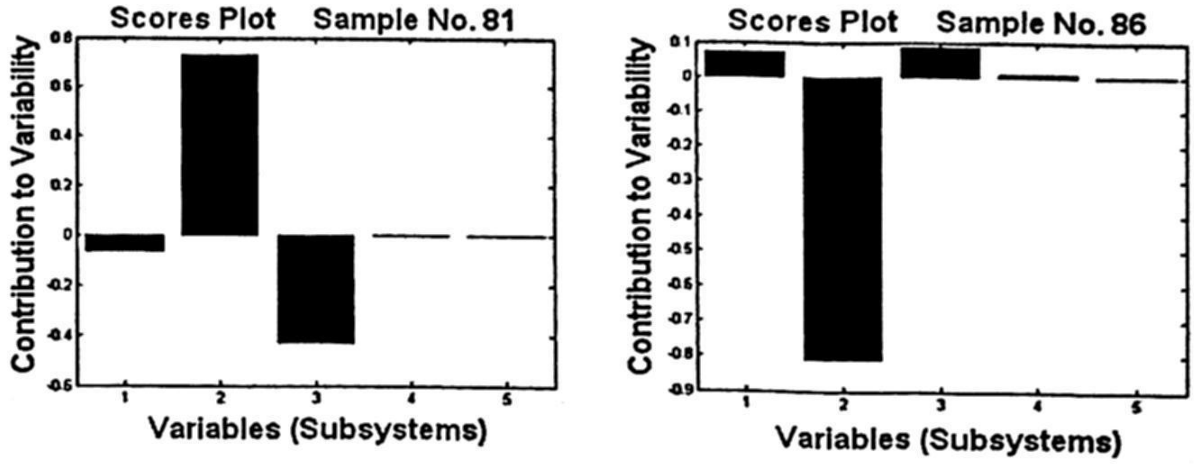


**Fig. 5.** Contribution charts for samples 81 and 86, indicating a probably problem in subsystem 2.

## 4.2 Multiple Fault

In the case of multiple fault, several runs have been carried out simulating a fault in two subsystems simultaneously. A ramp input in both subsystems, a ramp in one and a step in the other, a step in both, and a combination of a negative and positive ramp and step in one, the other or both. In this case contribution chart not always shows the real variables that possibly have problems. That is the reason explaining why the use of minimum and maximum limits for normal operation data plays an important role. As an example, a multiple fault is included in sample 80. Subsystem 3 having increments of 10% and subsystem 5 decrements of 0.9% in theirs corresponding electrical current normal operation value. Note that figure 6 depicts how the contribution chart does not show the real variables in multiple faulty mode, but it shows too how the implementation of statistic limits really does. Adittionally, the general performance of the proposal is shown in table 1.

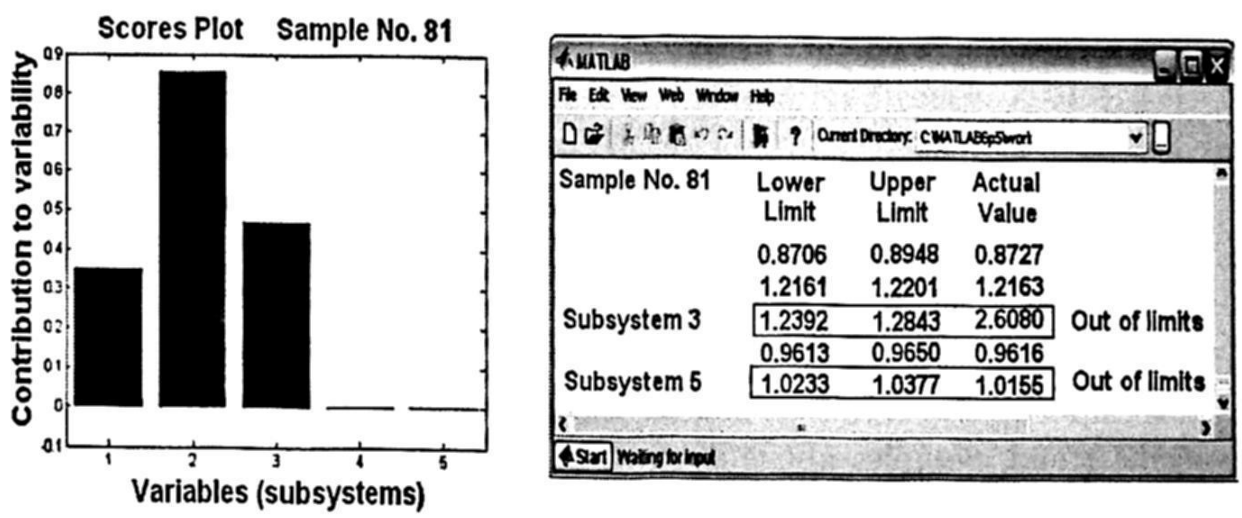


**Fig. 6.** Contribution charts for sample 81 (left) and the detection using the statiscal limits for the same sample (right).

**Table 1.** General performance of the proposal.

| Fault | Number of Simulations | Percentage Detected |
|---|---|---|
| Single | 50 | 98% |
| Multiple | 50 | 96% |

## 4.3 Measurement Noise

Table 2 shows the results for simulations taking into account different measurement noise magnitudes present in one, two or three subsystems. Figure 7 depicts the behaviour of control charts when it exists measurement noise of 0.1 magnitude present in subsystem 1.

**Table 2.** Performance of detection when measurement noise is present in one, two and three subsystems.

| Noise Magnitude | One Subs. | Two Subs. | Three Subs. |
|---|---|---|---|
| 0.0001 | 100% | 100% | 100% |
| 0.001 | 100% | 100% | 85% |
| 0.01 | 95% | 85% | 80% |
| 0.1 | 90% | 85% | 75% |

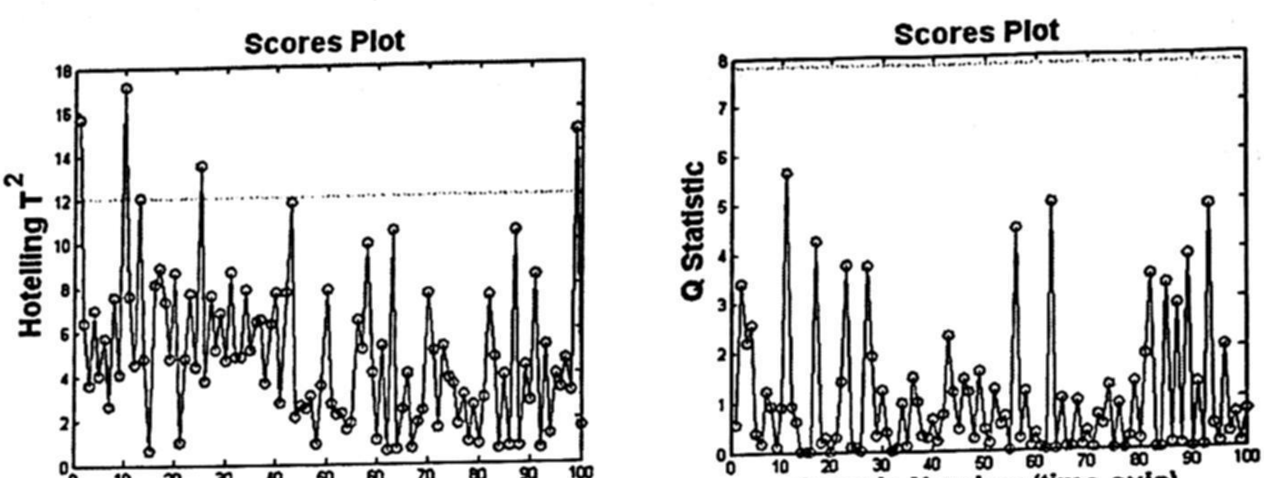


**Fig. 7.** Presence of measurement noise of 0.1 magnitude in subsystem 1.

It is important to note that none of the control charts shown in figure 7 has a specific trend. Samples above $T^2$ control chart limit are outliers. Note that all samples in $Q$ control chart are below its limit, which has sense because in this case noise does not brake the original correlation between variables. As $T^2$ is a measure of the multivariate distance between samples with respect to the center of data, it could detect a fault that keeps the correlation structure which could not be detected by $Q$. $Q$ detects faults that violate mass or energy balance pointing out a correlation breakdown.

### 4.4 Comparison against two similar frameworks

To observe the general performance of our proposal, a comparison against two similar Process History Based fault detection methods has been carried out. We have chossen [10] and [11] as the comparison frameworks because of their use of PCA and Multi-dimension features extraction of signal based on machine learning respectively. Table 3 shows this general performance comparison.

**Table 3.** Comparison of the general performance of our proposal against two similar frameworks.

| Detection of | PCA method | Machine Learning | Our Framework |
|---|---|---|---|
| Single Fault | √ | √ | √ |
| Multiple Fault | NO | √ | √ |
| Measurement Noise | NO | NO | √ |
| Process Noise | NO | NO | NO |
| Lack of Information | NO | NO | NO |

From table 3 it could be observed that PCA used as itself offers a poor data analysis. It is observed that machine learning based method (ML) as well as ours framework offer multiple fault detection. Nevertheless the use of ML needs to be implemented for each measured signal which generates a big quantity of data to be analysed. Meanwhile our framework avoids this data explosion by mean of the use of PCA, control charts and the obtention of the normal limits operation just for the statistical mean of variable been measured. An additional advantage over the other two frameworks is that ours detect measurement noise.

## 5 Conclusions

This paper has presented a fault detection framework based on history process data. An advantage over model based methods is that this framework only needs a good historical data set of normal system operation, which in practice it is relativelly easy to obtain for computer controlled industrial processes. This proposal is easy to implement and to adapt because when original process changes, it is only required to modify the original data base instead of develop a new mathematical model from it. Another advantage is that the use of PCA model allows to work with less quantity of data, but keeping the original correlation between variables. It is important to note that the use of $T^2$ and $Q$ control charts allows to distinguish between the presence of a fault and the presence of measurement noise. Also this framework could be used as an early way for fault detection as shown in subsection 4.1 when a deviation of 0.9% on variable been measured was detected. Finally the use of minimum and maximum limits for comparisons between a suspicious sample and its normal values gives the detection of a single or multiple faults existing in the system.

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